

Trigonometrik shaklda berilgan kompleks sonlarni ko'paytirish va bo'lish

Aytaylik, $z_1=r_1(\cos\varphi_1+i\sin\varphi_1)$ va $z_2=r_2(\cos\varphi_2+i\sin\varphi_2)$ kompleks sonlar berilgan bo'lsin. U holda, ularning ko'paytmasi

$$z_1 z_2 = (r_1 r_2)((\cos\varphi_1 \cos\varphi_2 - \sin\varphi_1 \sin\varphi_2) + i(\sin\varphi_1 \cos\varphi_2 + \sin\varphi_2 \cos\varphi_1))$$

bo'lib, trigonometriyadagi qo'shish teoremlariga asosan

$$z_1 z_2 = (r_1 r_2)(\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2)) \quad (11.3.1)$$

formulaga ega bo'lamiz. Bundan ko'rinadiki, trigonometrik shakldagi kompleks sonlarni ko'paytirish uchun modullarini ko'paytirish argumentlarini esa qo'shish kifoya ekan.

Xuddi shunga o'xshash, ularning bo'linamasi uchun

$$\frac{z_1}{z_2} = \left(\frac{r_1}{r_2} \right) (\cos(\varphi_1 - \varphi_2) + i\sin(\varphi_1 - \varphi_2))$$

formulani olish mumkin.

11.4. Kompleks sonni darajaga ko'tarish

Berilgan z kompleks sonning natural ko'rsatkichli n -darajasi z^n deb,

$$z^1 = z$$

ga, $2 \leq n \in \mathbb{N}$ bo'lganda,

$$z^n = z^{n-1} \cdot z$$

ga aytiladi.

Aytaylik, $z=r(\cos\varphi+i\sin\varphi)$ bo'lsin. U holda, (11.3.1) ga asosan

$$z^2 = z \cdot z = (r \cdot r)(\cos(\varphi + \varphi) + i\sin(\varphi + \varphi)) = r^2(\cos 2\varphi + i\sin 2\varphi),$$

$$z^3 = z^2 \cdot z = (r^2 \cdot r)(\cos(2\varphi + \varphi) + i\sin(2\varphi + \varphi)) = r^3(\cos 3\varphi + i\sin 3\varphi),$$

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$$z^n = r^n(\cos n\varphi + i\sin n\varphi) \quad (11.4.1)$$

ni olamiz. Bu kompleks sonni darajaga ko'tarish formulasidir ($n \in \mathbb{N}$).

Agar $|z|=r=1$ bo'lsa, (11.4.1) formuladan

$$(\cos\varphi + i\sin\varphi)^n = \cos n\varphi + i\sin n\varphi \quad (11.4.2)$$

ni olamiz. Buni *Muavr formulasi* deb yuritiladi.

Bu formulaning tatbiqlaridan biri sifatida $\cos n\varphi$ va $\sin n\varphi$ ni $\cos\varphi$ va $\sin\varphi$ lar orqali ifodalash formulalarini keltirish mumkin.

Haqiqatdan ham, (11.4.2) ning o'ng tomonidagi ikki hadning n darajasini Nyuton binomi formulasi bo'yicha yoyib, i ning darajalari bo'lgan

$$i^{4k-3}=i, \quad i^{4k-2}=-1, \quad i^{4k-1}=-i, \quad i^{4k}=1, \quad k \in \mathbb{N}$$

larni hisobga olib, kompleks sonlarning tenglik shartidan foydalansak, talab qilingan formulalarga ega bo'lamiz. Masalan, $n=4$ uchun

$$\begin{aligned} (\cos\varphi + i\sin\varphi)^4 &= \cos^4\varphi + i\sin^4\varphi + 4\cos^3\varphi \cdot i \sin\varphi + 6\cos^2\varphi \cdot i^2 \sin^2\varphi + 4\cos\varphi \cdot i^3 \sin^3\varphi + i^4 \sin^4\varphi = \\ &= \cos^4\varphi + i\sin^4\varphi - 4\cos^2\varphi \sin^2\varphi + 4i\cos\varphi \sin^3\varphi - \sin^4\varphi = \\ &= \cos^4\varphi + i(4\cos^3\varphi \sin\varphi - 6\cos^2\varphi \sin^2\varphi + 4\cos\varphi \sin^3\varphi) - \sin^4\varphi = \\ &= \cos^4\varphi + i\sin^4\varphi \end{aligned}$$

$$(\cos^4 \varphi - 6\sin^2 \varphi \cos^2 \varphi + \sin^4 \varphi) + i(4\cos^3 \varphi \cdot \sin \varphi - 4\cos \varphi \cdot \sin^3 \varphi) = \\ = \cos 4\varphi + i \sin 4\varphi.$$

Oxirgidan,

$$\cos 4\varphi = \cos^4 \varphi - 6\sin^2 \varphi \cos^2 \varphi + \sin^4 \varphi, \\ \sin 4\varphi = 4(\cos^3 \varphi \cdot \sin \varphi - \cos \varphi \cdot \sin^3 \varphi).$$

Bu yerda ham haqiqiy sonlar uchun darajaga ko'tarish amalining xossalari saqlanib qoladi. Undan tashqari,

$$(\bar{z})^n = \overline{(z^n)}$$

ham o'rinalidir.

11.5. Kompleks son dan ildiz chiqarish

z kompleks sonning n -darajali ($n \in \mathbb{N}$) ildizi $\sqrt[n]{z}$ deb shunday W kompleks songa aytiladiki, uning n -darajasi z ni beradi, ya'ni

$$W^n = z \quad (11.5.1)$$

bo'ladi.

Faraz qilaylik, $z = r(\cos \varphi + i \sin \varphi)$ trigonometrik shaklda berilgan bo'lsin.

$$W = \rho(\cos \psi + i \sin \psi)$$

bo'lsin deylik. (11.5.1) va (11.4.1) larga asosan

$$\rho^n (\cos n\psi + i \sin n\psi) = r(\cos \varphi + i \sin \varphi)$$

ni olamiz.

Endi, $z \neq 0$ bo'lganda, oxirgidan

$$\rho^n = r, \quad n\psi = \varphi + 2k\pi, \quad k \in \mathbb{Z}$$

kelib chiqadi. Oxirgi olingan tenglamalar haqiqiy sohadagi tenglamalardir. Ulardan

$$\rho = \sqrt[n]{r}, \quad \psi = \frac{\varphi + 2k\pi}{n}, \quad k \in \mathbb{Z}$$

larni, demak,

$$\sqrt[n]{z} = W_k = \sqrt[n]{r} \left(\cos \frac{\varphi + 2k\pi}{n} + i \sin \frac{\varphi + 2k\pi}{n} \right), \quad k = 0; 1; \dots; n-1 \quad (11.5.2)$$

formulani olamiz. Bu yerda k ning qolgan qiymatlarida sinus va kosinusning davriylik xossasi tufayli k ning yuqoridagi qiymatlarida olingan ildizlar takrorlanadi, ya'ni yangi ildiz qiymati kelib chiqmaydi. Demak, noldan farqli kompleks sonning n -darajali ildizi mavjud va u rosa n ta qiymatga ega bo'lar ekan. Kezi kelganda 0 ning natural ko'rsatkichli ildizi 0 ning o'zi bo'lib, faqat bitta qiymatga egaligini aytamiz.

1-misol. $\sqrt[3]{1}$ kompleks sohada hisoblansin.

Yechish. 1 ni trigonometrik shaklda yozamiz:

$$1 = \cos 0 + i \sin 0$$

Bundan $r=1$, $\varphi=0$ ni olamiz va ularni (11.5.2) ga qo'yib,

$$\sqrt[3]{1} = W_k = \cos \frac{2k\pi}{3} + i \sin \frac{2k\pi}{3} \quad k = 0; 1; 2$$

ni olamiz.

$$k=0 \Rightarrow W_0 = \cos 0 + i \sin 0 = 1,$$

$$k=1 \Rightarrow W_1 = \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i,$$

$$k=2 \Rightarrow W_2 = \cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i.$$

Demak, 1 ning kub ildizi kompleks sohada uchta qiymatga ega ekan, haqiqiy sohada esa faqat bitta 1 qiymatga egaligi bizga ma'lum.

2-misol. $\sqrt[4]{-1}$ ildiz kompleks sohada hisoblansin.

Yechish. Bu ildiz haqiqiy sohada mavjud emasligi ma'lum.

$$-1 = \cos \pi + i \sin \pi,$$

$$r=1, \quad \varphi=\pi.$$

Bularni (11.5.2) ga qo'yamiz:

$$\sqrt[4]{-1} = W_k = \cos \frac{\pi + 2k\pi}{4} + i \sin \frac{\pi + 2k\pi}{4}, \quad k = 0; 1; 2; 3.$$

$$W_0 = \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i = \frac{\sqrt{2}}{2}(1+i),$$

$$W_1 = \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i = \frac{\sqrt{2}}{2}(-1+i),$$

$$W_2 = \cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i = \frac{\sqrt{2}}{2}(-1-i),$$

$$W_3 = \cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i = \frac{\sqrt{2}}{2}(1-i).$$

3-misol. $\sqrt{1+i}$ ni toping.

Yechish.
$$r = \sqrt{1+i} = \sqrt{2}; \begin{cases} \sqrt{2} \cos \varphi = 1, \\ \sqrt{2} \sin \varphi = 1 \end{cases} \Rightarrow \varphi = \frac{\pi}{4};$$

$$1+i = \sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right).$$

$$\sqrt{1+i} = W_k = \sqrt[4]{2} \left(\cos \frac{\frac{\pi}{4} + 2\pi k}{2} + i \sin \frac{\frac{\pi}{4} + 2\pi k}{2} \right), \quad k = 0; 1.$$

$$W_0 = \sqrt[4]{2} \left(\cos \frac{\pi}{8} + i \sin \frac{\pi}{8} \right) = \sqrt[4]{2} \left(\frac{\sqrt{2+\sqrt{2}}}{2} + i \frac{\sqrt{2-\sqrt{2}}}{2} \right) =$$

$$= \frac{1}{\sqrt[4]{2}} \left(\sqrt{1+\frac{\sqrt{2}}{2}} + i \sqrt{1-\frac{\sqrt{2}}{2}} \right) \approx 1,0987 + 0,4551i;$$

$$W_1 = \sqrt[4]{2} \left(\cos \frac{9\pi}{8} + i \sin \frac{9\pi}{8} \right) = \frac{1}{\sqrt[4]{2}} \left(-\sqrt{1+\frac{\sqrt{2}}{2}} - i \sqrt{1-\frac{\sqrt{2}}{2}} \right) \approx -1,0987 - 0,4551i.$$

11. 6. Kompleks sohada ko'phad

Bu yerda n-darajali

$$P_n(z) = \sum_{i=0}^n A_i z^{n-i} \quad (11.6.1.)$$

ko'phadni qaraymiz. $n \in \mathbb{N}$; $A_0, A_1, \dots, A_n$ lar berilgan kompleks sonlar, ya'ni koeffitsientlar, $z=x+yi$ kompleks o'zgaruvchi bo'lib, $A_0 \neq 0$ deb faraz qilinadi (agar $A_0=0$ bo'lib qolsa, ko'phad darajasi n dan past bo'ladi).

$n=0$ bo'lgan holda ham ko'phad tushunchasi saqlab qolinib, uni nolinch darajali ko'phad deb yuritilib,

$$P_0(z)=A_0,$$

ya'ni o'zgaras deb qabul qilinadi.

Agar biror $z=z_0$ kompleks son uchun $P_n(z_0)=0$ bo'lsa z_0 kompleks son $P_n(z)$ ko'phadning ildizi deyiladi.

Undan tashqari, oliy algebra kursidan ma'lumki, A_0 ni bosh koeffitsient A_n ni esa ozod had deb ham yuritiladi. Shu bilan birga quyidagi algebraning asosiy teoremasi deb yuritiladigan teorema ham isbotlangandir.

11.6.1-teorema (Algebraning asosiy teoremasi). Har qanday natural n – darajali (11.6.1) ko'phad kamida bitta ildizga egadir [4].

Quyidagi Bezu teoremasi ham algebra kursidan ma'lum.

11.6.2-teorema (Bezu). Natural n –darajali (11.6.1) ko'phadni $z-a$ ikki hadga bo'lish natijasida chiqadigan qoldiq $P_n(a)$ ga tengdir, ya'ni

$$P_n(z)=(z-a)P_{n-1}(z)+R$$

bo'lsa, $R=P_n(a)$ dir, bu yerda $P_{n-1}(z)$ $(n-1)$ –darajali ko'phaddir.

Algebraning asosiy teoremasiga ko'ra $n \geq 1$ bo'lsa, (11.6.1) qandaydir z_1 ildizga egadir.

Bezu teoremasi asosida, agar z_1 (11.6.1) ko'phadning ildizi bo'lsa, $P_n(z_1)=0$ ekanligi sababli, uni

$$P_n(z)=(z-z_1)P_{n-1}(z)$$

ko'rinishda ko'paytuvchiga ajratish mumkinligi kelib chiqadi.

Agar $n \geq 2$ bo'lsa, $P_{n-1}(z)$ ham o'z navbatida qandaydir z_2 ildizga ega bo'ladi, u holda

$$P_{n-1}(z)=(z-z_2)P_{n-2}(z),$$

ya'ni

$$P_n(z)=(z-z_1)(z-z_2)P_{n-2}(z)$$

va hokazo, bu jarayonni davom ettirish natijasida n –so'ngi qadamda

$$P_n(z)=A_0(z-z_1)(z-z_2) \cdot \dots \cdot (z-z_n) \quad (11.6.2)$$

ga ega bo'lamiz. Bundan ko'rindiki, natural n –darajali ko'phad rosa n ta ildizga ega bo'lar ekan. Bu ildizlar ichida tenglari ham bo'lishi mumkin.

Ildizlari bo'yicha ko'phadni (11.6.2) ko'rinishda ifodalash uni ko'paytuvchilarga yoyish deb yuritiladi. Agar (11.6.2) yo'yilmada $(z-z_i)$ ko'paytuvchi k_i marta takrorlangan bo'lsa, z_i ni ko'phadning k_i karrali ildiz deb yuritiladi. Ildizlar karraliliklarini hisobga olgan holda (11.6.2) ni

$$P_n(z)=A_0(z-z_1)^{k_1} \cdot (z-z_2)^{k_2} \cdot \dots \cdot (z-z_q)^{k_q} \quad (11.6.3)$$

ko'rinishda yozish mumkin bo'lib, bu yerda $z_1, z_2, \dots, z_q$ lar ko'phadning bir-biridan farqli

ildizlari va $\sum_{i=1}^q k_i = n$ dir.

11.6.3–teorema. Agar n –darajali $P_n(z)$ ko‘phad aynan nolga teng, ya’ni z ning ixtiyoriy kompleks qiymatida nolga teng bo‘lsa, uning barcha koeffitsientlari nolga teng bo‘ladi.

Isbot. $P_n(z)=0$ bo‘lgani uchun ixtiyoriy n ta $z_1, z_2, \dots, z_n$ kompleks sonlarni uning ildizlari deb qarash mumkin, u vaqtda, (11.6.2) yoyilma asosida A_0 –bosh koeffitsient nolga tengligi, ya’ni $A_0=0$ bo‘lishi kelib chiqadi. Bu esa $P_n(z)$ n –emas ($n-1$)–darajali ekan degan xulosaga kelamiz va uning uchun ham

$$P_{n-1}(z)=A_1(z-z_1) \cdot \dots \cdot (z-z_{n-1})$$

yoyilmani yozib, $A_1=0$ ni va hokazo, bu jarayonni davom ettirib, $A_i = 0 \quad (i = \overline{0; n})$ ni olamiz.

11.6.4-teorema. Agar ikkita ko‘phad aynan bir-biriga teng bo‘lsa, ularning mos koeffitsientlari tengdir.

Isbot. $P_n(z)=Q_n(z)$ bo‘lsa, $R_n(z)=P_n(z)-Q_n(z)=0$ kelib chiqadi. $R_n(z)$ ning koeffitsientlari $P_n(z)$ va $Q_n(z)$ larning mos koeffitsientlari ayirmasidan iborat bo‘lib, ular 11.6.3-teoremaga ko‘ra nolga tengligidan $P_n(z)$ va $Q_n(z)$ larning mos koeffitsientlarining tengligi kelib chiqadi.

11.6.5-teorema. Agar n –darajali $P_n(z)$ ko‘phad $n+1$ ta turli $z_1, z_2, \dots, z_n, z_{n+1}$ nuqtalarda nolga teng bo‘lsa, u aynan nolga teng bo‘ladi.

Bu ham (11.6.2) yoyilma yordamida juda sodda isbotlanadi (o‘quvchiga havola qilamiz).

Agar (11.6.1) da $A_i (i=\overline{0, 1, \dots, n})$ koeffitsientlarning barchasi haqiqiy sonlar bo‘lsa, $P_n(z)$ ni *haqiqiy koeffitsientli ko‘phad* deb ataladi. Bunday ko‘phad mavhum $\alpha + \beta i$ ildizga ega bo‘lsa ($\beta \neq 0$), u qo‘shma $\alpha - \beta i$ ildizga ham ega bo‘lishini isbotlash mumkin.

Haqiqatdan ham, $A_i \in \mathbb{R} \Rightarrow \overline{A_i} = A_i \quad (i = \overline{0; n})$ ekanligini hisobga olsak,

$$P_n(\overline{z}) = \overline{P_n(z)}$$

bo‘lishini ko‘rish qiyin emas (bu kompleks sonlar ustida yuqorida ko‘rilgan amallarning xossalaridan kelib chiqadi). Shunday qilib, $\alpha + \beta i$ ($\beta \neq 0$) $P_n(z)$ ning ildizi bo‘lsa,

$$P_n(\alpha + \beta i) = 0$$

va

$$P_n(\alpha - \beta i) = \overline{P_n(\alpha + \beta i)} = \overline{P_n(\alpha + \beta i)} = \overline{0} = 0$$

kelib chiqadi.

Undan tashqari, $\alpha + \beta i$ $P_n(z)$ haqiqiy koeffitsientli ko‘phadning k karrali mavhum ildizi bo‘lsa, $(\alpha - \beta i)$ qo‘shma ildiz ham k karrali bo‘lishi kelib chiqadi. U holda, bunday mavhum ildizlarga mos (11.6.3) ning ko‘paytuvchilarini ajratib olsak,

$$\begin{aligned} (z - (\alpha + \beta i))^k (z - (\alpha - \beta i))^k &= ((z - (\alpha + \beta i))(z - (\alpha - \beta i)))^k = \\ &= (z^2 - 2\alpha z + (\alpha^2 + \beta^2))^k = (z^2 + pz + q)^k, \end{aligned}$$

ya’ni haqiqiy koeffitsientli kvadrat uchhadning k –darajasidan iborat ko‘paytuvchiga ega bo‘lamiz va bu uchhadning diskriminanti manfiyligiga ishonch hosil qilish qiyin emas.

Demak, $P_n(z)$ haqiqiy koeffitsientli ko‘phad bo‘lganda (11.6.3) yoyilmada $z - a$ ko‘rinishdagi chiziqli ko‘paytuvchilardan faqat haqiqiy ildizga moslari qolib, mavhum ildizga moslari esa ildizning qo‘shma juftiga mosi bilan ko‘payib haqiqiy koeffitsientli kvadrat uchhad ko‘rinishidagi ko‘paytuvchilarga aylanar ekan. Shunday qilib, bu hol uchun (11.6.3) ni quyidagicha yozish mumkin:

$$P_n(z) = A_0 \left(\prod_{i=1}^m (z - a_i)^{k_i} \right) \cdot \left(\prod_{j=1}^m (z^2 + p_j z + q_j)^{r_j} \right) \quad (11.6.4)$$

bu yerda $\sum_{i=1}^s k_i + 2 \sum_{j=1}^m r_j = n$ hamda barcha kvadrat uchhadlarning diskriminantlari manfiydir.

Shunday qilib, haqiqiy koeffitsientli natural darajali ixtiyoriy ko'phad (11.6.4) ko'rinishdagi chiziqli va manfiy diskriminantli kvadrat uchhadlardan iborat ko'paytuvchilar yoyilmasi kabi ifodalanar ekan. Bu yerda (11.6.4) yoyilmada faqat chiziqli yoki faqat kvadrat uchhad ko'rinishidagi ko'paytuvchilar qatnashgan bo'lishi ham mumkinligini aytamiz.

Misol. $P_4(z) = z^4 + 1$ ni ko'paytuvchilarga ajrataylik.

$$P_4(z) = (z^2 + 1)^2 - 2z^2 = (z^2 + 1)^2 - (\sqrt{2}z)^2 = (z^2 + \sqrt{2}z + 1)(z^2 - \sqrt{2}z + 1)$$

Demak, bu ko'phad yoyilmasida faqat kvadrat uchhadlar qatnashadi (qavslar ichidagi kvadrat uchhadlarning diskriminantlari manfiydir).

Demak, (11.6.4) yoyilmadagi chiziqli ko'paytuvchilar haqiqiy koeffitsientli ko'phadning haqiqiy ildizlariga, manfiy diskriminantli kvadrat uchhadlardan iborat ko'paytuvchilar esa uning qo'shma mavhum ildizlariga mos kelar ekan.

11.6.6-teorema. Agar $P_n(z)$ ko'phad m karrali z_0 ildizga ega bo'lsa ($m \leq n$), bu z_0 son ko'phadning $(m-1)$ – tartibligacha bo'lgan hosilalari uchun ham ildiz bo'lib, $P_n^{(m)}(z_0) \neq 0$ bo'ladi.

Isbot. Yuqoridagi mulohazalardan, agar z_0 $P_n(z)$ ko'phadning m karrali ildizi bo'lsa,

$$P_n(z) = (z - z_0)^m P_{n-m}(z)$$

o'rinli bo'lib, bunda $P_{n-1}(z)$ – qandaydir $(n-1)$ - darajali ko'phad va $P_{n-m}(z)$ – qandaydir $(n-m)$ - darajali ko'phad va $P_{n-m}(z_0) \neq 0$ dir. Oxirgi tenglikni $m-1$ marta differensiallab,

$$P_n^{(m-1)}(z) = m \cdot (m-1) \cdot \dots \cdot 2(z - z_0) \cdot P_{n-m}(z) + (z - z_0)^2 \cdot Q_{n-m-1}(z) \quad (11.6.5)$$

ni olamiz, bu yerda $Q_{n-m-1}(z)$ $(n-m-1)$ –darajali qandaydir ko'phaddir. Oxirgida $z = z_0$ desak, $P_n^{(m-1)}(z_0) = 0$ bo'lishi kelib chiqadi. Undan tashqari, $k \leq m-1$ bo'lganda $P_n^{(k)}(z)$ k –tartibli hosiladan iborat ko'phad $(z - z_0)^{m-k}$ ko'paytuvchiga ega bo'lishiga ishonch hosil qilish qiyin emasdir va shu sababli $0 \leq k \leq m-1$ bo'lganda $P_n^{(k)}(z_0) = 0$ dir.

Endi (11.6.5) ni differensiallab,

$$P_n^{(m)}(z) = m! P_{n-m}(z) + (z - z_0) Q_{n-m-2}(z)$$

ga ega bo'lamiz, bu yerda $Q_{n-m-2}(z)$ $(n-m-2)$ –darajali qandaydir ko'phaddir. Oxirgida $z = z_0$ deb,

$$P_n^{(m)}(z_0) = m! P_{n-m}(z_0) \neq 0$$

ni olamiz. Teorema isbotlandi.

Eslatma. Agar $z - z_0$ $P_n(z)$ ko'phadning ko'paytuvchilari yoyilmasida birinchi darajada qatnashsa, ya'ni $P_n(z_0) = 0$, $P_n'(z_0) \neq 0$ bo'lsa, z_0 ni $P_n(z)$ ko'phadning *oddiy ildizi* (ba'zan bir karrali ildizi) deb yuritiladi.

11-bobga doir mashqlar

1. Kompleks sonni trigonometrik shaklda yozing:

$$1). i; \quad 2). \frac{-1+i}{1+i}; \quad 3). \frac{1}{-2+i} - \frac{1}{2+i}; \quad 4). \frac{2+i}{1-i} \cdot \frac{2-i}{-1+i}.$$

2. Berilgan tengsizlik kompleks tekislikda nimani tasvirlaydi?

$$1). |z-2| < 2; \quad 2). 1 < |z| < 2; \quad 3). \frac{\pi}{4} < \arg z < \frac{\pi}{3}.$$

3. Kompleks sohada ildiz chiqarish amalini bajaring.

$$1). \sqrt{-4}; \quad 2). \sqrt[3]{-8}; \quad 3). \sqrt[4]{16}; \quad 4). \sqrt[6]{64}.$$

4. Kompleks sohada tenglamani yeching:

$$1). x^2 + 2x + 5 = 0; \quad 2). x^4 + 2x^2 + 5 = 0.$$

5. Berilgan ko'phadni ko'paytuvchilarga ajrating:

$$1). x^3 + 3x^2 - 4; \quad 2). x^4 + 4x^2 + 5; \quad 3). x^4 + 6x^2 - 7.$$

11-bob bo'yicha o'z bilimingizni sinab ko'ring

1. Mavhum birlik haqida tushuncha bering.

2. Kompleks songa ta'rif bering.

3. Kompleks sonlar ustidagi asosiy arifmetik amallarni ta'riflang.

4. Kompleks sonning geometrik talqini nimadan iborat?

5. Kompleks sonning moduli va argumenti haqida tushuncha bering.

6. Kompleks sonning trigonometrik, ko'rsatkichli va algebraik shakllarini yozing.

7. Kompleks sonni darajaga ko'tarish amalini ta'riflang. Muavr formulasini yozing.

8. Kompleks sohada ildiz chiqarish amali qanday bajariladi?

9. Ko'phad kompleks sohada qanday aniqlanadi?

10. Algebraning asosiy teoremasini keltiring.

11. Bezu teoremasini keltiring.

12. Haqiqiy koeffitsientli ko'phad qanday ko'paytuvchilarga ajraladi?